MAT1325 : HW#5 DUE MARCH 28, 2012

Instructions: The work you submit should be well-organized, well-justified and written legibly. Calculations alone, without minimum explanation to make it readable, are insufficient. Bonus questions can earn bonus points. The rest of the assignment is worth 35 points.


2. Consider the equation $Ax^2 + By^2 = C$, for $A, B, C \in \mathbb{R}$ fixed and variables $x$ and $y$. The constants $A$, $B$ and $C$ can be positive, negative or zero, leading to 27 different cases. For each case, we want to understand the set $S_{A,B,C} = \{(x, y) \in \mathbb{R}^2 \mid Ax^2 + By^2 = C\}$.

Make a table of all the different cases, identifying in each case: the type of $S_{A,B,C}$ (e.g. point, empty set, ellipse, hyperbola, . . .); the $x$ and $y$ intercepts, if any; and in the case of a hyperbola, the asymptotes.

Hint: the case $(A, B, C)$ is identical to $(-A, -B, -C)$. Other cases are equivalent by swapping the roles of $x$ and $y$, if you’d like to use that to reduce the size of your table.

3. Stewart, Chapter 9.6 page 681 # 22. Hint: complete the square. For your sketch: first identify the point on which the graph is centered (which is the origin for equations in standard form but not for this example).


5. Suppose that $\{x_n\}_{n \in \mathbb{N}}$ and $\{y_n\}_{n \in \mathbb{N}}$ are sequences of real numbers. Prove that

$$\lim_{n \to \infty} x_n = a \quad \text{and} \quad \lim_{n \to \infty} y_n = b$$

if and only if the sequence of vectors $\{(x_n, y_n)\}_{n \in \mathbb{N}}$ converges to the vector $(a, b)$. Hint: the string of inequalities you proved on the first homework may be useful!

6. (Bonus) Recall that $||(x, y)|| = \sqrt{x^2 + y^2}$ represents the length or magnitude of the vector $(x, y) \in \mathbb{R}^2$. Recall also that the difference of two vectors is calculated componentwise: $(x, y) - (a, b) = (x - a, y - b)$. The $\varepsilon$, $\delta$ definition of the continuity of a function of two variables is as follows:

Date: March 17, 2012.
A function \( f(x, y) \) is continuous at a point \((a, b)\) in its domain \(D\) if for every \( \varepsilon > 0 \) there exists a \( \delta > 0 \) such that for every \((x, y) \in D\) such that
\[
\|(x, y) - (a, b)\| < \delta
\]
we have
\[
|f(x, y) - f(a, b)| < \varepsilon.
\]
Prove using this definition that the function \( f(x, y) = x^2 + y^2 \) is continuous at any \((a, b) \in \mathbb{R}^2\).

7. Stewart, Section 11.2 page 755 # 10, 16, 24.

8. Stewart, Section 11.3 page 767–8 # 10, 26, 34, 46, 56.